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Energy levels and far-infrared spectroscopy for two electrons in a semiconductor nanoring

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Abstract

The effects of electron-electron interaction of a two-electron nanoring on the energy levels and far-infrared spectroscopy have been investigated based on a model calculation which is performed within the exactly numerical diagonalization. It is found that the interaction changes the energy spectra dramatically, and also shows significant influence on the far-infrared spectroscopy. The crossings between the lowest spin-singlet and triplet states induced by the coulomb interaction are clearly revealed. Our results are related to the experiment recently carried out by A. Lorke *et al.* [Phys. Rev. Lett. **84**, 2223 (2000)].

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I. INTRODUCTION

Rapid progress in nanostructure technology has made it possible to fabricate various nanometer quantum devices which have potential applications in microelectronics. Such ultrasmall devices contain only a few electrons and the electron-electron interaction is proposed to be of great importance to theirs energy level structures and optical properties. It leads to a number of new quantum phenomena. One of the most interesting phenomena is the *spin oscillation* of the ground state in an external magnetic field, which is due to the interplay between three energies: the confinement potential, the Zeeman energy, and the electron-electron interaction. The simplest case, a pillar few electrons quantum dot (QD) with a parabolic potential, has been extensively investigated, and the spin oscillation in a QD is well understood.[1−3] The theoretical predictions are also confirmed in the experiment through the conductance measurements in the finite drain-source voltage regime.[4]

The semiconductor quantum ring is another interesting example. In 1993, D. Mailly *et al.* measured the *persistent currents* in a mesoscopic single GaAs ring induced by a magnetic flux threading the interior of ring,[5] and the experimental results have attracted a lot of theoretical interests. One of the basic questions addressed by many theoretical explanation is concerned with the role of the electron-electron interaction.[6−13] For a *narrow-width* rings, an adiabatic approximation allows one to decouple the radial motion from the angular motions and to arrive at analytical solutions for the wave functions and the energy spectra.[12,14] As shown by L. Wendler *et al.*,[12,14] the interplay of the Coulomb repulsion between the electrons and the confining potential forms a relatively rigid rotator with internal azimuthal excitations and confined radial motions, *i.e.*, the picture of a rotating Wigner molecule. However, at the moment, the influence of the electron-electron interaction in a *finite-width* and *nanoscopic* quantum ring, *i.e.*, nanoring, is still less well understood.[15−17]

Very recently, using the self-assembly techniques, A. Lorke and collaborators demonstrate the realization of nanoscopic semiconductor quantum rings inside a completed field-effect transistor (FET) structure.[18−20] Quite different from the conventional sub-micron meso-

scopic structures, the nanorings are in the *true* quantum limit. By applying two complementary spectroscopic techniques, capacitance-voltage (CV) spectra and far-infrared (FIR) spectroscopy, they investigate both the ground state transition and excitation's properties of these two-electron nanorings in a magnetic field perpendicular to the plane of rings. Although the main experimental results can be qualitatively explained by the single electron picture, some contradictions remain, i.e. the coulomb interaction energy estimated roughly 20 meV, is too large to be ignored safely. Hence, more in-depth theoretical works are desirable, especially in view of the existences of the very strong electron-electron coulomb interaction.

In this paper, we would like to study the energy levels and FIR spectroscopy of a two-electron nanoring, and pay special attention to the effects of the Coulomb interaction. First of all, for a ring-like confinement potential, the total Hamiltonian cannot be separated into the center-of-mass and the relative-motion terms. We develope a new theoretical method to handle this non-separability. It consists of the well-known series solution method,[2,21] which is effective to solve the single particle problem, and the exact diagonalization method.[22,23] On the other hand, we show that the electron-electron interaction can change the energy levels significantly. An obvious feature induced by the interaction is the intersection between the lower levels. It presents the spin oscillation of the ground state in an external field. Moreover, the results obtained by A. Lorke *et al.* can be explained more realistically in our model. Further, the generalized Kohn theorem breaks down in the FIR spectroscopy due to the mixing between the center-of-mass and the relative-motion modes as mentioned above.[24,25] We outline here two prominent features of the FIR spectroscopy caused by electron-electron interaction: splitting and dis-continuous drops of the resonance energies, which are suggested to be detectable with circularly polarized far-infrared light.

The organization of this paper is as follows. In the next section, we introduce the model and Hamiltonian briefly. In Sec III, the main procedures of the numerical calculation are outlined. Sec IV is devoted to the results and discussions, followed by a summary in Sec V.

II. THE MODEL AND HAMILTONIAN

The nanoring is considered to have two electrons with an effective conduction-band-edge mass m_e^* moving in the $x-y$ plane, and a ring-like confining potential can be introduced $U(\vec{r}) = \frac{1}{2}m_e^*\omega_0^2(r-R_0)^2$,[17–19] where ω_0 is the characteristic frequency of the radial confinement and R_0 is the ring's radius. This system is subjected to a perpendicular uniform magnetic field which is described by a vector potential $\vec{A}(\vec{r}) = \frac{1}{2}\vec{B}\times\vec{r}$ in the symmetric (or circular) gauge. The resulting Hamiltonian is given by

$$\mathcal{H} = \sum_{i=1,2}\left\{\frac{1}{2m_e^*}\left(\vec{p}_i + e\vec{A}(\vec{r}_i)\right)^2 + U(\vec{r}_i)\right\} + \frac{e^2}{4\pi\varepsilon_0\varepsilon_r|\vec{r}_1 - \vec{r}_2|}, \tag{1}$$

where $\vec{r}_i = (x_i, y_i)$ and $\vec{p}_i = -i\hbar\vec{\nabla}_i$ are respectively the position vector and momentum operator of the i-th electron with charge $-e$. ε_0 is the vacuum permittivity and ε_r is the static dielectric constant of the host semiconductor. In addition, there also exist the spin interaction term with the magnetic field $\mathcal{H}_{spin} = g^*\mu_B\left(\vec{S}_1 + \vec{S}_2\right)\cdot\vec{B}$, where g^* is the Landé factor, and μ_B is the Bohr magneton. Due to the small g^* in semiconductor, in general, $\mathcal{H}_{spin}$ is very small and can be ingored safely (*i.e.*, for $g^* = -0.44$ in GaAs materials and $B = 10$ T, the typical value of $\mathcal{H}_{spin}$ is 0.25 meV). It is worthy to note that in the limit case $R_0 = 0$, the nanoring simply reduces to a parabolic quantum dot, which can be solved trivially by separating the Hamiltonian into center-of-mass and relative-motion terms. The appearance of R_0 breaks this separability, and it becomes more complex.

We apply the exact diagonalization method by constructing the basis with the single particle wavefunctions of Hamiltonian

$$\mathcal{H}_s = \frac{1}{2m_e^*}\left(\vec{p} + e\vec{A}(\vec{r})\right)^2 + U(\vec{r}). \tag{2}$$

These wavefunctions labelled by the radial quantum number n and orbital angular-momentum quantum number m have the form

$$\psi_{nm}(\vec{r}) = R_{nm}(r)\exp(im\varphi) \qquad n = 0, 1, 2..., \quad m = 0, \pm 1, ..., \tag{3}$$

where the radial part R_{nm} will be solved exactly by using the series expansion method.[2]

III. FORMULA AND CALCULATION METHODS

A. Series solution

For the sake of convenience, we use the effective atomic units, in which the effective Rydberg $R_y^* = \frac{m_e^* e^4}{2\hbar^2 (4\pi\varepsilon_0\varepsilon_r)^2}$ and the effective Bohr radius $a^* = \frac{4\pi\varepsilon_0\varepsilon_r\hbar^2}{m_e^* e^2}$ are taken to be the energy and length units, respectively. Then, the Hamiltonian (2) has the form

$$\mathcal{H}_s = -\vec{\nabla}^2 + \frac{1}{4}\gamma_b^2 r^2 + \frac{1}{4}\gamma_d^2 (r - R_0)^2 + \gamma_b \hat{L}_z, \tag{4}$$

where the magnetic field γ_b is measured in the unit $\frac{\hbar\omega_c}{2R_y^*}$ with cyclotron frequency $\omega_c = \frac{eB}{m_e^*}$, $\gamma_b \hat{L}_z$ is the Zeeman term, and $\gamma_d^{-1/2} = \left(\frac{R_y^*}{\hbar\omega_0}\right)^{1/2}$ is related to confinement region of the electrons. It is interesting to note how large the units of semiconductor materials are. For GaAs materials, for example, $R_y^* = 5.8$ meV, $a^* = 10$ nm, and $\gamma_b = 1$ corresponds to $B = 6.75$ T.

Now, we have to solve the Schrödinger-like equation

$$\mathcal{H}_s \left[R_{nm}(r) \exp\left(im\varphi\right)\right] = E_{nm} \left[R_{nm}(r) \exp\left(im\varphi\right)\right] \tag{5}$$

to obtain the energy E_{nm} and radial part of wavefunction $R_{nm}(r)$. It is easy to find the equation satisfied by the function $R_{nm}(r)$

$$\left\{\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \left[(E_{nm} - m\gamma_b) - \frac{m^2}{r^2} - \frac{1}{4}\gamma_b^2 r^2 - \frac{1}{4}\gamma_d^2 (r - R_0)^2\right]\right\} R_{nm}(r) = 0. \tag{6}$$

We are prevented from analytically exact solutions of the eigenvalue problem because Eq. (6) with suitable boundary conditions is beyond the analytical problem of confluent hypergeometric equations. However, we can use the method of series expansion to obtain the series forms in different regions of Eq. (6) and the exact values of E_{nm}.[2,21]

It should be noted that $r = 0$ and $r = +\infty$ are respectively the regular and irregular singularity points of Eq. (6). So it is natural to divide the whole region $[0, +\infty)$ into three parts, $[0, +\infty) = [0, r_0) \cup [r_0, r_\infty) \cup [r_\infty, +\infty)$, where r_0 and r_∞ are the two dividing points.

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With suitable adjustment of r_0 and r_∞, very high numerical precision can be archived. In all the three kinds of regions, the function $R_{nm}(r)$ is found to be the form:

$$R_{nm}(r) = \begin{cases} Ar^l \sum\limits_{n=0}^{\infty} a_n r^n & 0 \leqslant r < r_0, \\ C_i \sum\limits_{n=0}^{\infty} c_{in} (r - R_i)^n + D_i \sum\limits_{n=0}^{\infty} d_{in} (r - R_i)^n & R_i \leqslant r < R_{i+1}, \\ B \exp\left(-\frac{1}{4}\gamma r^2 + \frac{\gamma_d^2}{2\gamma} R_0 r\right) r^s \sum\limits_{n=0}^{\infty} b_n r^{-n} & r_\infty \leqslant r < +\infty, \end{cases} \tag{7}$$

where $\gamma = \sqrt{\gamma_b^2 + \gamma_d^2}$, $l = |m|$ and $s = \frac{(E_{nm} - m\gamma_b)}{\gamma} - 1 - \frac{\gamma_b^2 \gamma_d^2}{4\gamma^3} R_0^2$. In order to improve precision, we further divide the region $[r_0, r_\infty)$ into N pieces, denoted by R_i $(i = 1, ..., N)$, here $R_1 = r_0$ and $R_N = r_\infty$. A, C_i, D_i and B are constants, a_n, c_{in}, d_{in} and b_n are expanding coefficients and can be determined by the recurrence relation coming from Eq. (6), the initial values of coefficients are chosen to be $a_0 = 1$, $c_{i0} = d_{i1} = 1$, $c_{i1} = d_{i0} = 0$ and $b_0 = 1$.

Using the matching conditions at $r = R_i$ $(i = 1, ..., N)$, and the 2×2 transfer matrices, we can deduce the equation for eigenenergies E_{nm} easily. Then the constant A, C_i, D_i and B can be evaluated by normalization condition, and $R_{nm}(r)$ is obtained numerically.

To close this subsection, it is interesting to point out that the method mentioned above is very suitable for numerical calculations, and can be modified to handle various differential equations similar to Eq. (6).

B. Exact diagonalization

Once the single particle wavefunctions are obtained, we go ahead to construct the basis of the two-electron wavefunctions. It is obvious that the total z component of the angular momentum operator of two electrons, $\hat{L}_{z,total} = \hat{L}_{z1} + \hat{L}_{z2}$, is a constant of motion, i.e., $\left[\mathcal{H}, \hat{L}_{z,total}\right] = 0$ is valid, from which the rotational invariance of the problem follows. Thus, one can work in one subspace labelled by a good quantum number L instead of the whole huge Hilbert space. We denote the corresponding two-electron wavefunction by $\psi_L(\vec{r}_1 \sigma_1; \vec{r}_2 \sigma_2)$ which depends on the spatial coordinates $\{\vec{r}_i\}$ and the spin coordinates $\{\sigma_i\}$ $(i = 1, 2)$. Because the Hamiltonian of Eq. (1) does not depend on the spin operator, in our two

electrons case, the wavefunction can separate into the orbital part $\psi_L\left(\vec{r}_1, \vec{r}_2\right)$ and the spin part $\chi_S\left(\sigma_1, \sigma_2\right)$:

$$\psi_{LS}\left(\vec{r}_1\sigma_1; \vec{r}_2\sigma_2\right) = \psi_L^{\lambda_1\lambda_2}\left(\vec{r}_1, \vec{r}_2\right)\chi_S\left(\sigma_1, \sigma_2\right) \tag{8}$$

with

$$\psi_L^{\lambda_1\lambda_2}\left(\vec{r}_1, \vec{r}_2\right) = c\left[\psi_{n_1m_1}\left(\vec{r}_1\right)\psi_{n_2m_2}\left(\vec{r}_2\right) + (-)^S\psi_{n_2m_2}\left(\vec{r}_1\right)\psi_{n_1m_1}\left(\vec{r}_2\right)\right], \tag{9}$$

where the normalized wavefunction $\psi_{LS}\left(\vec{r}_1\sigma_1; \vec{r}_2\sigma_2\right)$ is further labelled by the total spin number $S = 0$ or 1, corresponding to the singlet and triplet state. λ_i $(i = 1, 2)$ stands for the quantum number pair (n_im_i). $L = m_1 + m_2$. $\psi_{nm}\left(\vec{r}\right)$ is the single particle wavefunction. $c = \sqrt{\frac{1}{2}}$ as $(n_1m_1) \neq (n_2m_2)$, and $c = \frac{1}{2}$ as $(n_1m_1) = (n_2m_2)$. Obviously, the wavefunction constructed above satisfies the antisymmetric condition,

$$\mathcal{P}_{12}\psi_{LS}\left(\vec{r}_1\sigma_1; \vec{r}_2\sigma_2\right) = -\psi_{LS}\left(\vec{r}_1\sigma_1; \vec{r}_2\sigma_2\right), \tag{10}$$

where $\mathcal{P}_{12}$ is the permutation operator.

In the next step, we diagonalize the Hamiltonian (1) numerically in a restricted configuration space. The space is constructed by choosing the wavefunctions having the form (8) in the lowest f levels. The secular equation of finite degree f is given by

$$\det\left\|\left(E_i^{(0)} - E\right)\delta_{ij} + \Delta_{ij}\right\| = 0, \qquad i, j = 1, ..., f, \tag{11}$$

where $E_i^{(0)} = E_{n_1^im_1^i} + E_{n_2^im_2^i}$ is the single particle energy, $i = \left(n_1^im_1^i; n_2^im_2^i\right)$ and $j = \left(n_1^jm_1^j; n_2^jm_2^j\right)$ represent the quantum number levels. Δ_{ij} is the matrix element of electron-electron interaction in unit of R_y^*

$$\Delta_{ij} = \left\langle i\left|\frac{2}{|\vec{r}_1 - \vec{r}_2|}\right|j\right\rangle = \Delta_{ij}^c + (-)^S\Delta_{ij}^e, \tag{12}$$

where

$$\Delta_{ij}^c = \int\int d\vec{r}_1 d\vec{r}_2\psi_{n_1^im_1^i}^*\left(\vec{r}_1\right)\psi_{n_2^im_2^i}^*\left(\vec{r}_2\right)\frac{2}{|\vec{r}_1 - \vec{r}_2|}\psi_{n_1^jm_1^j}\left(\vec{r}_1\right)\psi_{n_2^jm_2^j}\left(\vec{r}_2\right), \tag{13}$$

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and

$$\Delta_{ij}^e = \int \int d\vec{r}_1 d\vec{r}_2 \psi_{n_1^i m_1^i}^* (\vec{r}_1) \psi_{n_2^i m_2^i}^* (\vec{r}_2) \frac{2}{|\vec{r}_1 - \vec{r}_2|} \psi_{n_2^j m_2^j} (\vec{r}_1) \psi_{n_1^j m_1^j} (\vec{r}_2). \qquad (14)$$

Δ_{ij}^c, Δ_{ij}^e can be computed numerically from the series solution of $\psi_{nm}(\vec{r})$.

By diagonalizing the secular equation (11) in each subspace (S, L), we obtain the m-th energy level E_m and the corresponding two-electron wave-function:

$$\Phi_m (\vec{r}_1 \sigma_1; \vec{r}_2 \sigma_2) = \sum_{\lambda_1 \lambda_2} A_{\lambda_1 \lambda_2}^m \psi_L^{\lambda_1 \lambda_2} (\vec{r}_1, \vec{r}_2) \chi_S (\sigma_1, \sigma_2). \qquad (15)$$

C. Optical absorption

In the electronic dipole approximation,[14] the absorption coefficient is given by

$$\alpha (\omega) = c\omega \sum_{fi} \left| \left\langle f \left| \vec{e} \cdot \vec{d} \right| i \right\rangle \right|^2 \delta (\omega - \omega_{fi}) \left(f_i^{(0)} - f_f^{(0)} \right), \qquad (16)$$

where $\vec{e}$ is the complex polarization vector of the spatially constant external electronic field, $\vec{d} = -(\vec{r}_1 + \vec{r}_2)$ is the electronic dipole operator of two electrons, and $f^{(0)}$ is the equilibrium Fermi distribution. The summation is over all the two-electron eigenstates, ω_{fi} is proportional to the energy difference between the initial state $| i \rangle$ and final state $| f \rangle$: $\omega_{fi} = \omega_f - \omega_i = \left(\frac{E_f - E_i}{\hbar} \right)$. c is a constant factor. Restricting ourselves to zero temperature $T = 0K$, Eq. (16) reduces to

$$\alpha (\omega) = c\omega \sum_{f} \left| \left\langle f \left| \vec{e} \cdot \vec{d} \right| 0 \right\rangle \right|^2 \delta (\omega - \omega_{f0}), \qquad (17)$$

where $| 0 \rangle$ and $| f \rangle$ represent the ground state and excited state respectively.

For circularly polarized light we have $\vec{e} = \sqrt{\frac{1}{2}} (1, \pm i)$, from which it follows $\vec{e} \cdot \vec{d} = -\sqrt{\frac{1}{2}} (r_1 e^{\pm i\varphi_1} + (1 \rightarrow 2))$. Using Eq. (15), we obtain

$$\left\langle f \left| \vec{e} \cdot \vec{d} \right| 0 \right\rangle = -\sqrt{\frac{1}{2}} \sum_{\lambda_1^0 \lambda_2^0} \sum_{\lambda_1^f \lambda_2^f} A_{\lambda_1^f \lambda_2^f}^{f*} A_{\lambda_1^0 \lambda_2^0}^0 \left(\left(d_{\lambda_1^0 \lambda_1^f}^{\pm} \delta_{\lambda_2^0 \lambda_2^f} + (-)^S d_{\lambda_1^0 \lambda_2^f}^{\pm} \delta_{\lambda_2^0 \lambda_1^f} \right) + (1 \rightarrow 2) \right), \quad (18)$$

where the single particle matrix elements $d_{\lambda\lambda'}^{\pm}$ are defined by

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$$d_{\lambda\lambda'}^{\pm} = \langle \lambda' | r \exp(\pm i\varphi) | \lambda \rangle = \delta_{m\pm 1,m'} \int_0^\infty r^2 R_{\lambda'}(r) R_\lambda(r) \, dr \tag{19}$$

and $|\lambda\rangle$ represents the single particle wave-function $\psi_{nm}(\vec{r})$. It is obvious that the absorption coefficient satisfies the dipole selection rule $\Delta L = \pm 1$. Substituting Eq. (18) into Eq. (17) and taking

$$\delta(\omega - \omega_{f0}) = \frac{\Gamma/\pi}{(\omega - \omega_{f0})^2 + \Gamma^2}, \tag{20}$$

where Γ is a phenomenological broadening parameter, we arrive at

$$\alpha^{\pm}(\omega) = c \frac{\omega}{2} \sum_f \left| \sum_{\lambda_1^0 \lambda_2^0} \sum_{\lambda_1^f \lambda_2^f} A_{\lambda_1^f \lambda_2^f}^{f*} A_{\lambda_1^0 \lambda_2^0}^0 \left(\left(d_{\lambda_1^0 \lambda_1^f}^{\pm} \delta_{\lambda_2^0 \lambda_2^f} + (-)^S d_{\lambda_1^0 \lambda_2^f}^{\pm} \delta_{\lambda_2^0 \lambda_1^f} \right) + (1 \to 2) \right) \right|^2 \times$$
$$\frac{\Gamma/\pi}{(\omega - \omega_{f0})^2 + \Gamma^2}, \tag{21}$$

where $\pm$ corresponds to right and left circularly polarized lights, respectively.

To check the numerical accuracy of the calculations we have used the f-sum rules for the dipole operators, which can be expressd in terms of ground-state quantities:[26,27]

$$\int_0^\infty \left(\alpha^+(\omega) + \alpha^-(\omega) \right) d\omega = \langle 0 | \left[\left(\vec{e} \cdot \vec{d} \right)^+, \left[\mathcal{H}, \left(\vec{e} \cdot \vec{d} \right) \right] \right] | 0 \rangle = N \tag{22}$$

in the effective atomic units, where c is taken to be 1, and $N = 2$ is the number of electrons.

It is important to point out that in our case, due to non-separability of the center-of-mass and the relative-motion modes, the generalized Kohn theorem, which means that FIR can only be used to excite the center-of-mass modes of electrons parabolically confined in circular quantum dot, will not be held further. Thus we expect our FIR absorption result may reflect an excitation of the relative motion of two electrons.

IV. RESULTS AND DISCUSSIONS

To explain the experimental measurements,[18] we have taken the material parameters $\varepsilon_r = 12.4$ and $m_e^* = 0.067 m_e$ for GaAs in our calculations. The radial confinement strength

$\hbar\omega_0$ and the ring's radius R_0 are chosen to be 12 meV and 14 nm respectively. The corresponding width of ring is about 15 nm, which means that the electrons are confined in a wide ring. Thus in contrast to the rotating Wigner molecule picture in a narrow-width quantum ring,[12,14] the more pronounced energy spectra and optical properites are expected. For the calculation in each (S, L) subspace, we first solve the single particle problem and save several hundreds single particle states, then pick up the suitable single particle states to construct thousands of two-electron states, among which only the lowest f energy levels are selected. Here we simply point out that our numerical diagonalization scheme is very efficient and essentially exact in the sense that the accuracy can be improved as desired by increasing f. For instance, for the ground state in $(0, 0)$ subspace, the use of 64 basis states allows the precision to be within the relative convergence of $\sim 10^{-4}$. On the other hand, by checking the f-sum rule, we find that the relative error of our FIR calculation is less than 1%.

A. Spin oscillation

The energy levels of two electrons in a nanoring as a function of the magnetic field have been plotted in Fig.1. As mentioned above, only the total spin and total angular momentum are conserved in our model, and then for the sake of clearness, we only plot the lowest level in each (S, L) subspace. In order to understand the role of the electron-electron interaction in the two-electron spectra better, we first describe the characteristics of the energy levels of two electrons in the nanoring without interaction. As shown in Fig. 1(a), for small magnetic field, the spectra have the characteristics of disk-like quantum dot, since the electrons are confined in a wide nanoring whose radius is comparable to its effective width. As the magnetic field increases from zero, there are minima for the states with negative angular momentum L. These minima are caused by the interplay between the Zeeman term and the ring-like confinement potential. Moreover, the level with $L = 0$ increases monotonically, and changes more dramatically than the others. This leads to a ground state transition from

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$L = 0$ to $L = -2$ around $B = 8$ T, which also reflects the fact that the nanoring becomes more and more narrow with increasing the magnetic field. In the sufficient high field regime, it is quite safe to speculate that the levels will be the same as the levels in one-dimensional ring under a uniform magnetic field.

The electron-electron interaction can significantly change the characteristics of the spectra described above. As shown in Fig. 1(b), clearly visible is the ground state transitions around $B = 3$, 7 and 10 T. These transitions are quite different from that mentioned in the previous paragraph: not only the total angular momentum but also the total spin are changed. They present the spin-singlet-spin-triplet oscillation of the ground state in the magnetic field, i.e., $(0, 0) \rightarrow (1, -1) \rightarrow (0, -2) \rightarrow (1, -3)$ states and so on. This phenomenon is indeed qualitatively similar to that seen in a quantum dot.[2]

For a better understanding of the singlet-triplet oscillation, it is interesting to study the electron-electron interaction energies E_r, defined by the difference between the energy E with interaction and $E^{(0)}$ without interaction. In Fig. 2, the E_r for different states are plotted as a function of the magnetic field. It is readily seen that the E_r increase with the magnetic field and the ordering is as follows: $E_r(0, -1) > E_r(0, 0) > E_r(0, -3) > E_r(1, -2) > E_r(1, 0) > E_r(1, -1) > E_r(0, -2) > E_r(1, -3)$.... Compared with the corresponding results in quantum dot,[2] we find that the ordering depends significantly on the form of the confinement potential, e.g., the ordering $E_r(0, 0) > E_r(0, -1)$ in quantum dot is reversed in our case. However, the ordering $E_r(0, 0) > E_r(1, -1) > E_r(0, -2) > E_r(1, -3)$ is still preserved, thus the trivial crossover around $B = 8$ T in Fig. 1(a) moves to the position $B = 3$ T in Fig. 1(b).

We comment on some other effects caused by the interaction. (i) When the electron-electron interaction is excluded, the states $(0, L)$ and $(1, L)$ ($L = -1, -3$) are degenerate in the whole regime of the magnetic field. This degeneracy can be understood by the fact that those states are constructed with the same single particle states, the only difference between them is spin part which have no influence to energy levels in the absence of interaction. When the electron-electron interaction turns on, the whole energy levels are shifted to high

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energies due to the repulsive coulomb interaction and the degenerated levels split. (ii) In the high magnetic field regime, the energy levels appear to be more separated in the presence of interaction, and it indicates that the interaction effect becomes larger in this regime.

B. FIR spectroscopy

In the previous subsection, we have demonstrated that the electron-electron interaction can induce some transitions of the ground state. It may be possible to observe these transitions in the lower energy optical absorption. Now, we present the FIR spectroscopy to elucidate such transitions. Here, the phenomenological broadening parameter Γ is assumed to be 0.5 meV.

In Fig. 3, we plot the FIR absorption for the circularly polarized lights as a function of the magnetic field. Compared with the no interaction case, where the right and left circularly polarized (labeled by $\sigma_\pm$) resonance energies are roughly given by $\omega_\pm = \frac{1}{2}\left(\sqrt{\omega_c^2 + 4\omega_0^2} \pm \omega_c\right)$ in the low field (as shown in Fig. 4, σ_+ and σ_- are represented by the square and filled circle symbols, respectively), we find some unusual features in the presence of interaction. (i) As the magnetic field increases, the lowest right circularly polarized σ_+ resonance energy shows discontinuous drops around 3, 7 and 10 T (see Fig. 3a). In contrast, this behavior is only found at $B \approx 8$ T in the case without the Coulomb interaction (see Fig. 4). Naturally, the discontinuous drops are originated from the spin oscillation of the ground state as mentioned above. Note that they are less reflected in the σ_- case. (ii) The absorption peak around 20 meV splits into three or more subpeaks, especially in the σ_- polarization case. The striking behavior can be understood from the mixing of the center-of-mass and the relative-motion modes. Certainly, it has to keep in mind that due to the mixing there are many irregularly spaced and near-degenerated energy levels hybridized from the relative-motion mode in each (S, L) subspace. The splitting can be identified as a transition from the ground state to those "hybrid" states, showing its two-electron characteristic. (iii) For both circular polarizations, the lowest resonance energy shows slight blue shift due to the Coulomb interaction.

Motivated by the above prominent features, we suggest that the spin transitions of the ground state and the other effects induced by the electron-electron interaction can be observed with circularly polarized light. However, as pointed out by A. Lorke, in theirs experiment, the single particle states are quite accurate basis for the description of the measured FIR resonance. It seems in contrast to ours expectation. We argue here that the controversy comes from the resolution of the experiment. With low resolution, one is difficult to distinguish the details of the FIR spectroscopy discussed above, and only the profile is explored. As shown in Figs. (3a) and (3b), we can indeed observe that the profile of the FIR resonance can be described by the single particle picture. Therefore, we expect that our predictions about the electron-electron interaction may be confirmed by a high resolution experiment in the future.

V. SUMMARY

In conclusion, we have investigated the energy levels and far-infrared spectroscopy of a two-electron nanoring in a magnetic field. Because of the electron-electron interaction as well as the interplay between the magnetic field and ring-like confinement potential, the nanoring exhibits rich electronic structures. An obvious feature induced by the interaction is the intersection between the lower levels. It presents the spin oscillation of the ground state on the magnetic field. i.e., $(0,0) \rightarrow (1,-1) \rightarrow (0,-2) \rightarrow (1,-3)$ states and so on. This phenomenon is indeed qualitatively similar to that seen in a quantum dot, suggesting its intrinsic nature of zero-dimensional quantum structures.

The profile of the FIR spectroscopy is roughly captured by the single particle picture as indicated by a recent experiment.[18] However, the ring-like confinement potential doesn't allow the application of the generalized Kohn theorem. Thus the single particle picture is inadequate for seeing any effect due to electron-electron interaction. We have outlined here two prominent features of the FIR spectroscopy caused by electron-electron interaction: splitting and dis-continuous drops of the resonance energies. We suggest that those two

features can be detectable by using the circularly polarized far-infrared light with high experimental resolution.

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Figures Captions

Fig. 1. The energy levels of two electrons in a nanoring are plotted as a function of the magnetic field with $\hbar\omega_0 = 12$ meV and $R_0 = 14$ nm in the (a) absence and (b) presence of electron-electron interaction. The spin singlet and triplet states are labelled by solid and dashed lines, respectively. The quantum number of each state (total spin, total angular momentum) are also indicated. Note that only the lowest energy level of each (S, L) subspace is selected.

Fig. 2. The net Coulomb energies are plotted as a function of the magnetic field. The other parameters are the same as in Fig. 1.

Fig. 3. A logarithmic 3D plot of the far-infrared absorption coefficient as a function of the magnetic field for (a) right and (b) left circularly polarized light.

Fig. 4. Far-infrared absorption resonance energies in the absence of electron-electron interaction as a function of the magnetic field. The square and filled circle symbols are corresponding to right and left circularly polarized lights, respectively.

(a)

(0,−3) (1,−3)

(1,0)

(1,−2)

(0,−2)

(0,−1) (1,−1)

(00)

(b)

(0,−3)

(1,−3)   (1,0)

(1,−2)

(0,−2)

(0,−1)

(1,−1)

(0,0)

(a)

(b)